More on Hypothesis Testing

There are 4 outcomes of a hypothesis test:

	H_0 is true	H_A is true
You reject H_0	Error (Type I)	✓
You fail to reject H_0	✓	Error (Type II)

α and β

- The significance level of a test is α, the probability of making Type I error
  α=P[reject H_0 given that H_0 is true]
- β is the probability of making Type II error:
  β=P[fail to reject H_0 given that H_A is true]
- The power of a test is 1-β, ie.
  Power = P[reject H_0 given H_A is true]

Example: find α, β and Power

You have two coins: one lands on heads 85% of the time, the other lands on heads 35% of the time. You pick one coin and flip it: if it lands heads you decide it is the 85% coin, tails you decide it is the 35% coin.

- H_0: p=.85, H_A: p=.35
- α=P[reject H_0 | H_0 is true] = P[tails|p=.85] = .15
- β=P[fail to reject H_0 | H_A is true] = P[heads|p=.35]=.35
- Power = 1-β = .65

Relationship Between Error Types

- Typically in order to lower the probability of Type I error you end up increasing Type II error

H_0: defendant is not guilty, H_A: is guilty

If you require stronger evidence to convict, that will lower probability of Type I error (convicting innocent people), but you will also increase the probability that the guilty will be set free.

- **The only way to lower both types of errors is to increase your sample size**

Example: foul shots

A basketball player has a 70% foul-shot accuracy rate. He practices during the off season and tells the coach he's improved to 85%. The coach tests the player – if he can make at least 9 out of 10, he believes him.

- H_0: p=.70, H_A: p=.85
- α= P(Type I error) = P(Player makes at least 9 shots despite 70% rate) = 1-binomcdf(10,.70,8)=.1493
- β=P(Player makes 8 or fewer with 85% rate) =binomcdf(10,.85,8)=.4557
- Power = 1-β= 1-.4557= .5443